Question – 2 : Dimensional Analysis of Time of Fall

Using only dimensional analysis, derive an expression for the time it takes an object to fall from a height h under gravity g.
Assume no air resistance and that the only variables involved are the drop height h and the gravitational acceleration g.
Do not use known kinematic formulas derive the relationship purely from unit analysis.

Solution:

We aim to determine the fall time t based solely on dimensional reasoning using the variables:

- h (height): [L]

- g (gravitational acceleration) : $\left[\frac{L}{T^2}\right]$

Assume the relationship:

$t = C \cdot h^a \cdot g^b$

Where C is a constant (to be determined experimentally) and a, b are exponents to be solved through dimensional consistency.

The dimensional equation becomes:

$$[T] = [L]^a \left(\frac{[L]}{[T]^2}\right)^b = L^{(a+b)}.T^{(-2b)}$$

Now equate the dimensions on both sides:

Time: $-2b = 1 \rightarrow b = -1/2$

Length: $a + b = 0 \rightarrow a = 1/2$

Therefore, the fall time is directly related to the square root of the ratio between height and gravitational acceleration.

In physics, the complete formula is: $t = \sqrt{\frac{2h}{g}}$

The constant $\sqrt{2}$ is not predicted by dimensional analysis and must be determined through experiment.